

Adaptive Modulation with Impulsive Interference

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Abstract—In this letter, we analyze power and rate adaptation in a point-to-point link with Rayleigh fading and impulsive interference. We model the impulsive interference as a Bernoulli-Gaussian random process. Adaptation is used to maximize the average spectral efficiency by changing power and rate of the transmission subject to an average power and instantaneous probability of error constraints. Without impulsive interference, it is well known that water-filling is optimal for block fading. We provide two simple schemes that show that the conventional water-filling algorithm is not optimal in an impulsive interference channel.

I. INTRODUCTION

In current wireless systems, frequency planning and medium access control protocols help regulate co-channel interference. Despite these protocols, due to the size, complexity of the networks and device imperfections, interference is a major impediment. Sometimes, this interference is impulsive in nature [1]. For example, in a cellular system impulsive interference occurs because of partial loading and channel feedback instability. Also, current wireless networks are interfered by other devices such as microwave ovens that are in close by vicinity and operate in the same frequency band [2] and the interference generated is impulsive in nature. In such scenarios, Bernoulli-Gaussian arrival process is used to model the impulsive interference process [3]. Impulsive noise exists in power line communications and has been analysed a lot in powerline literature in past decade. Zimmermann et al. proposed the multipath model for a power line channel [4]. They also analysed and modeled impulsive noise that occurs in power-line communications [5]. Meng et al. gave a frequency domain approach to model and analysed the effects of noise on broadband power-line communications [6]. In [7], impulsive interference is analyzed in an OFDM system for power-line communications, but rate and power adaptation are not considered. In [8], rate and power adaptation are considered in the context of power-line communications. However the analysis is carried out with an average interference power assumption.

Present day wireless systems adapt transmission rate and transmit power to achieve a target probability of error, subject to an average power constraint. In a block fading channel with additive white Gaussian noise, the signal-to-noise-ratio (SNR) is constant across a coherence time interval (coherence time interval is the duration when the channel gain remains constant). Hence a water-filling based solution (based on the SNR fed back at the beginning of every coherence time interval)

is optimal to adapt the power and the transmission rate [9] in such channels.

Adaptive modulation in the presence of AWGN and persistent interference has been studied in [10], [11]. In a two user down-link interference channel, when both the channels experience Rayleigh fading, it was shown in [10] that binary power allocation performs better than water-filling when ratio of the gain of the interfering path to the gain of the direct path is greater than a threshold. In [11] it was shown that though binary power allocation is sub-optimal for a system with multiple interfering links, it is preferred as it is computationally simple. However, there has been little understanding of how to adapt the power and modulation rate even in a point-to-point link when the interference is impulsive in nature.

II. SYSTEM MODEL

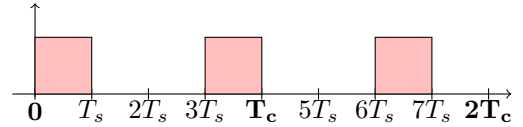


Fig. 1: Bernoulli interference process: We consider $N = 4$ symbols in a coherence time interval. The interference occurs as a block of T_s with probability p . The channel estimate of the first symbol of coherence time interval is fed back based on which the transmitter adapts its power and transmission rate.

In this letter, we consider a fading channel with impulsive interference. We assume that the duration of interference T_i is same as that of symbol period T_s , i.e., $T_i = T_s$ as shown in Figure 1. We consider the following discrete time system model for Bernoulli impulsive interference [12],

$$y_m = h_m x_m + w_m + i_m, \quad \forall m \geq 0. \quad (1)$$

Here y_m is the received signal, x_m is the transmitted M-QAM symbol, w_m denotes a zero mean Gaussian random variable of variance σ_n^2 and i_m denotes the interference process. The fading denoted by h_m is an i.i.d process. We assume that the magnitude of the fading $|h_m|$ is Rayleigh distributed and hence $H_m = |h_m|^2$ is exponentially distributed. Without loss of generality, the mean of H_m is assumed to be one. We will use H to denote a generic instance of H_m and it remains constant for the coherence time interval T_c . The average signal power (\bar{P}) is also assumed to be one. The interference process

is given by

$$i_m = b_m g_m,$$

where b_m is Bernoulli process and g_m is a sequence of i.i.d. Gaussian random variables of variance σ_i^2 . The Bernoulli process b_m is an i.i.d sequence of zeros and ones with $\mathbb{P}(b_m = 0) = 1 - p$ and $\mathbb{P}(b_m = 1) = p$. The interference process is shown in Figure 1. The receiver feeds back SINR to the transmitter at the beginning of every coherence time interval.

The SINR of symbols not affected by impulsive interference is $\frac{H\bar{P}}{\sigma_n^2}$. As H is exponentially distributed of unit mean and average signal power (\bar{P}) is assumed to be one, SINR is exponentially distributed of mean

$$\bar{\gamma} = \frac{1}{\sigma_n^2},$$

and its PDF is denoted as $f_n(\gamma)$. Similarly SINR of symbols affected by impulsive interference is exponentially distributed with mean

$$\frac{1}{\sigma_n^2 + \sigma_i^2} = \frac{\bar{\gamma}}{(1 + \mu)},$$

where $\mu = \sigma_i^2/\sigma_n^2$ and the corresponding PDF is denoted as $f_i(\gamma)$. So the SINR distribution is $(1 - p)f_n(\gamma) + pf_i(\gamma)$.

III. ADAPTIVE MODULATION IN THE PRESENCE OF IMPULSIVE INTERFERENCE

The objective is to adapt transmit rate and power instantaneously to maximize average spectral efficiency while keeping the instantaneous probability of error below a threshold P_b and satisfying the power constraint over a long horizon. Here we focus on M-QAM modulation. In [9], the probability of bit error with M-QAM constellation can be approximated as

$$P_b(\gamma) = ce^{\frac{-1.5\gamma}{M-1}}, \quad (2)$$

where $c=0.2$ and γ is the SINR, where $\gamma = \frac{H\bar{P}}{\sigma^2}$, H is the channel power, \bar{P} is average symbol power and σ^2 is noise power. Let $P(\gamma)$ and $M(\gamma)$ denote the transmit power and modulation scheme chosen based on the SINR fed back at the beginning of the coherence time interval. On adaptation, the SINR is $\frac{HP(\gamma)}{\sigma^2}$ which is equivalent to $\frac{\gamma P(\gamma)}{\bar{P}}$. So the probability of error is

$$P_b(\gamma) = ce^{\frac{-1.5\gamma P(\gamma)}{(M(\gamma)-1)}}. \quad (3)$$

Hence for a target P_b , rearranging (3) gives

$$M(\gamma) = 1 + \frac{k\gamma P(\gamma)}{\bar{P}},$$

where $k = -1.5/\ln(\frac{P_b}{c})$. We want to maximize the average spectral efficiency $\mathbb{E}_\gamma[\log_2 M(\gamma)]$ subject to an average power constraint $\mathbb{E}_\gamma[P(\gamma)] \leq \bar{P}$.

When SINR is constant in a coherence time interval, water-filling is the optimal power adaptation scheme [9] and is given by,

$$\frac{kP(\gamma)}{\bar{P}} = \begin{cases} \frac{1}{\eta} - \frac{1}{\gamma}; & \gamma \geq \eta \\ 0; & \gamma < \eta, \end{cases} \quad (4)$$

where the threshold η is obtained by solving

$$\int_{\eta}^{\infty} (\eta^{-1} - \gamma^{-1}) g(\gamma) d\gamma = k. \quad (5)$$

The average spectral efficiency is given by

$$\mathbb{E}_\gamma[\log_2 M(\gamma)] = \int_{\eta}^{\infty} \log_2 \left(\frac{\gamma}{\eta} \right) g(\gamma) d\gamma, \quad (6)$$

where $g(\gamma)$ denotes the PDF of the SINR in the coherence interval.

But optimal power and rate adaptation for a channel where SINR varies within the coherence time interval is not known in literature to the best of our knowledge. We apply the conventional water-filling algorithm discussed above in an impulsive interference system and compare with two other algorithms to show that conventional water-filling is not optimal with impulsive interference.

In this letter ‘outage’ is defined as the event when the probability of error of symbol transmitted is greater than the fixed probability of error constraint P_b .

A. Conventional water-filling

The average spectral efficiency achieved by conventional water-filling in an impulsive interference system is not same as given by (6), where $g(\gamma) = (1 - p)f_n(\gamma) + pf_i(\gamma)$. This is because all the symbols in a coherence time interval do not experience the same SINR. Hence the rate and power that are adapted using the SINR of first symbol of a coherence time interval can not satisfy the probability of error constraint across all the symbols in a coherence time interval.

Theorem 1. *The average spectral efficiency achieved by conventional water-filling in a channel affected by impulsive interference is*

$$(1 - p)^2 \int_{\eta}^{\infty} \log_2 \left(\frac{\gamma}{\eta} \right) f_n(\gamma) d\gamma + p \int_{\eta}^{\infty} \log_2 \left(\frac{\gamma}{\eta} \right) f_i(\gamma) d\gamma.$$

Proof:

Let the instantaneous SINR of symbols not affected by impulsive interference and affected by impulsive interference in a coherence time interval be denoted as γ_n and γ_i respectively. So $\gamma_i = \gamma_n/(1 + \mu)$, where $\mu = \sigma_i^2/\sigma_n^2$. Let the first symbol of a coherence time interval not be affected by impulsive interference as illustrated in Fig. 1 in the time interval $[T_c, 5T_s]$. If water-filling is utilized, the symbols in the coherence time interval $([T_c, 2T_c])$ are transmitted at power $P(\gamma_n) = \frac{\bar{P}}{k}(\frac{1}{\eta} - \frac{1}{\gamma_n})$ and rate $\log_2(M(\gamma_n)) = \log_2(\gamma_n/\eta)$. So the probability of error of a symbol affected by impulsive interference in this block (for example $[6T_s, 7T_s]$ in Fig. 1) on adaptation is

$$\begin{aligned} P_b(\gamma_i) &= ce^{\frac{-1.5\gamma_i P(\gamma_n)}{P(M(\gamma_n)-1)}} \\ &= ce^{\frac{-1.5\gamma_i(\frac{1}{\eta} - \frac{1}{\gamma_n})}{k(\frac{\gamma_n}{\eta} - 1)}} \\ &= ce^{\frac{-1.5}{k(1+\mu)}} > P_b. \end{aligned}$$

As $P_b(\gamma_i) > P_b$, this results in outage. Hence, outage occurs when the first symbol in a coherence slot is not affected by impulsive interference and symbols within that coherence time interval are affected by impulsive interference. As the arrival process is memoryless, the outage probability is $P_o = p(1-p)$. So the average spectral efficiency is

$$\begin{aligned} & ((1-p) - p(1-p)) \int_{\eta}^{\infty} \log_2 \left(\frac{\gamma}{\eta} \right) f_n(\gamma) d\gamma + \\ & p \int_{\eta}^{\infty} \log_2 \left(\frac{\gamma}{\eta} \right) f_i(\gamma) d\gamma, \quad (7) \end{aligned}$$

where η is obtained from (5). ■

We now consider two other alternative transmission strategies: The first strategy is to be aggressive and assume that impulsive interference is not present *i.e.*, all symbols are affected by AWGN of variance σ_n^2 and adapt power and rate accordingly. The second alternative is to be conservative and assume that impulsive interference exists always *i.e.*, all symbols are affected by AWGN of variance $\sigma_n^2 + \sigma_i^2$ and base the adaptation on this assumption.

B. Aggressive water-filling

In this method we neglect the impulsive interference and water-filling is used assuming only the noise density function. However we use H instead of the SINR for adaptive modulation¹. Since the interference and noise are zero mean processes, the channel H can be estimated by the receiver by averaging out the sequence of training symbols that the transmitter sends at the beginning of a coherence interval. This channel value H is fed back to the transmitter and is used to adapt power and rate for the coherence interval.

As in Section III, on power and rate adaptation, the probability of error is $P_b = ce^{\frac{-1.5HP(H)}{\sigma_n^2(M(H)-1)}}$. The objective is to maximize the average spectral efficiency $\mathbb{E}_H[\log_2 M(H)]$, where $M(H) = 1 + \frac{kHP(H)}{\bar{P}}$, subject to an average power constraint

$$\int_0^{\infty} P(H) f(H) dH \leq \bar{P},$$

where $k = -\frac{1.5\bar{P}}{\sigma_n^2 \ln(\frac{P_b}{c})}$ and $f(H)$ is the PDF of H . The solution to the above problem is similar to the water-filling solution in Section III. The water-filling threshold α_n is obtained by solving $\int_{\alpha_n}^{\infty} (\frac{1}{\alpha_n} - \frac{1}{H}) f(H) dH = k$. Similar to the proof of Theorem 1, it can be seen that by using the threshold α_n for symbols affected by impulsive interference, will result in an outage as the probability of error constraint will be violated. Hence the average spectral efficiency achieved by aggressive water-filling is

$$R_n = \int_{\alpha_n}^{\infty} (1-p) \log_2 \left(\frac{H}{\alpha_n} \right) f(H) dH.$$

¹Due to space constraint we omit the proof that in a Rayleigh faded channel both the approaches give the same average spectral efficiency

Using the fact that $f(H) = \exp(-H)$, we obtain

$$\int_{\alpha_n}^{\infty} \log_2 \left(\frac{H}{\alpha_n} \right) f(H) dH = \log_2(e) \left(\frac{e^{-\alpha_n}}{\alpha_n} - k \right),$$

where $k = -\frac{1.5\bar{P}}{\sigma_n^2 \ln(\frac{P_b}{c})}$. Hence

$$R_n = (1-p) \log_2(e) \left(\frac{e^{-\alpha_n}}{\alpha_n} - k \right). \quad (8)$$

C. Conservative water-filling

In this method, we assume that the symbols are always affected by impulsive interference. As we assume impulsive interference exists always, the objective is to satisfy an instantaneous probability of error constraint $P_b = ce^{\frac{-1.5HP}{(\sigma_n^2 + \sigma_i^2)(M-1)}}$. As seen in previous Section, water-filling threshold α_i can be found from average power constraint, $\int_{\alpha_i}^{\infty} (\frac{1}{\alpha_i} - \frac{1}{H}) f(H) dH = k$, where $k = -\frac{1.5\bar{P}}{(\sigma_n^2 + \sigma_i^2) \ln(\frac{P_b}{c})}$. As we are assuming the worst case scenario, outage does not occur and hence the average spectral efficiency achieved by conservative water-filling is

$$R_i = \int_{\alpha_i}^{\infty} \log_2 \left(\frac{H}{\alpha_i} \right) f(H) dH. \quad (9)$$

$$= \log_2(e) \left(\frac{e^{-\alpha_i}}{\alpha_i} - k \right), \quad (10)$$

where $k = -\frac{1.5\bar{P}}{(\sigma_n^2 + \sigma_i^2) \ln(\frac{P_b}{c})}$.

IV. NUMERICAL RESULTS

For simulation, we generate exponential random variables (square of Rayleigh) with mean SNR for $1-p$ fraction of total symbols (one lakh symbols). Similarly p fraction of total symbols are generated as exponential random variables with mean $\frac{\text{SNR}}{1+\mu}$. Using these generated values, adaptive modulation algorithms (conventional, conservative and aggressive water-filling) as described in the previous sections are evaluated to verify that the average spectral efficiency obtained is same as derived in equations (7), (8) and (10). In practice, only discrete rate modulation can be implemented. In the subsequent figures, the curves obtained through evaluating the algorithms by generating the random variables are termed as “WF simul” and the curves obtained by evaluating the equations are termed as “WF theory”. The discrete rate modulation as it is only a minor modification of the scheme described in the previous Sections and is described in detail in [14] for the AWGN case.

The theoretical and simulated rate curves for conventional (7), aggressive (8) and conservative (10) water-filling schemes are plotted in Figure 2 and 3 for low μ , SNR and high μ , SNR case. We first observe that the simulations match the theoretical expressions. At low p , *i.e.*, low impulsive interference, we see that the aggressive approach of neglecting interference in computing the water-filling threshold is better. On the other hand, at high p , we see that it is better off to always assume persistent interference in computing the

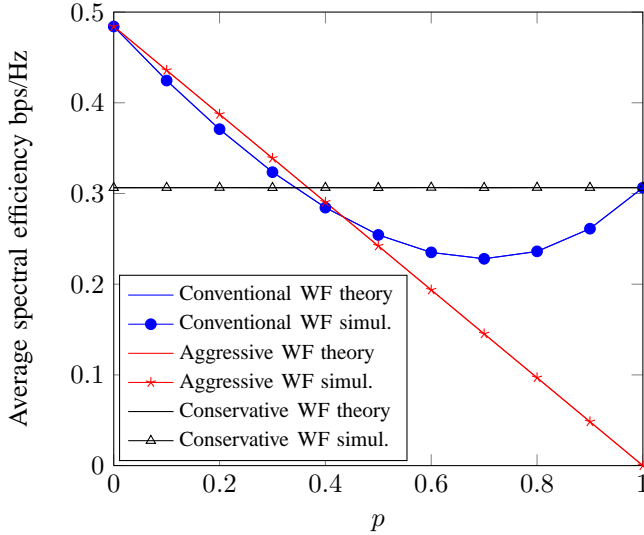


Fig. 2: $\mu = \sigma_i^2/\sigma_n^2 = 0$ dB, SNR = 0 dB.

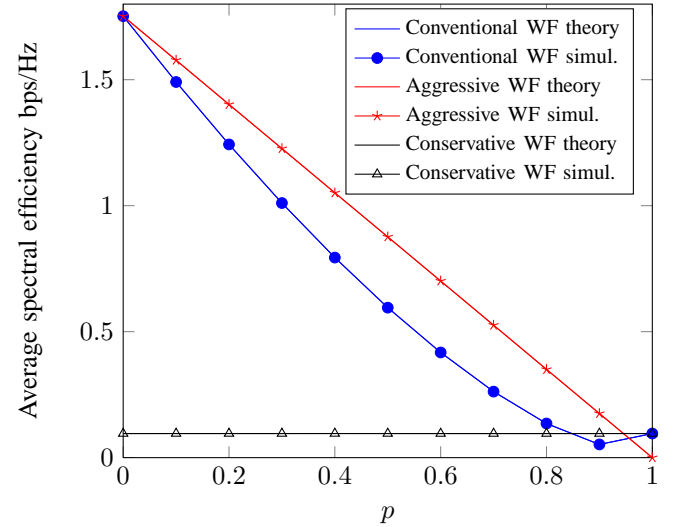


Fig. 3: $\mu = \sigma_i^2/\sigma_n^2 = 20$ dB, SNR = 10 dB

threshold. We also observe that the aggressive or conservative schemes are always better than conventional water-filling *i.e.*, $\max(R_n, R_i) > R_w \forall p \in (0, 1)$. As there are no analytical closed form expressions to calculate water-filling thresholds η , α_n and α_i it is tough to analytically compare the average spectral efficiency achieved by conventional water-filling (R_w) with the ones achieved by aggressive (R_n) and conservative (R_i) water-filling. These graphs (as counter examples) prove that conventional water-filling is not the optimal scheme with impulsive interference even when the interference is uncorrelated across time. Aggressive and conservative schemes are optimal when $p = 0$ and $p = 1$ respectively. The intuition to use aggressive and conservative schemes was to check if these could be better than conventional water filling at low and high values of p respectively. But an interesting observation is that for all values of p either of these two schemes is better than conventional water-filling.

Let p_{th} be the intersection point (p) of the aggressive and conservative achievable average rates. Also we observed that p_{th} increases with μ . This is because when impulsive interference power is high, the achievable average rate due to conservative scheme (10) is very low. Hence for a wide range of p values it is better to experience outage and transmit aggressively than to transmit conservatively.

V. CONCLUSION

In this letter, we have derived the average spectral efficiency of a Bernoulli-Gaussian impulsive interference channel achieved by three different water-filling schemes subject to an average power and instantaneous probability of error constraint. We observe that the conventional water filling is sub-optimal. In fact, even simpler schemes that neglect interference (for computing water filling threshold) outperform conventional water filling solution for a range of parameters.

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